OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4725

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Turn over

1 Express $\dfrac{2+3\text{i}}{5-\text{i}}$ in the form $x + \text{i}y$, showing clearly how you obtain your answer. **[4]**

2 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ a & 5 \end{pmatrix}$. Find

(i) $\mathbf{A}^{-1}$, **[2]**

(ii) $2\mathbf{A} - \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$. **[2]**

3 Find $\displaystyle\sum_{r=1}^{n}(4r^3 + 6r^2 + 2r)$, expressing your answer in a fully factorised form. **[6]**

4 Given that $\mathbf{A}$ and $\mathbf{B}$ are 2×2 non-singular matrices and $\mathbf{I}$ is the 2×2 identity matrix, simplify

$$\mathbf{B}(\mathbf{AB})^{-1}\mathbf{A} - \mathbf{I}.$$ **[4]**

5 By using the determinant of an appropriate matrix, or otherwise, find the value of k for which the simultaneous equations

$$\begin{aligned} 2x - y + z &= 7, \\ 3y + z &= 4, \\ x + ky + kz &= 5, \end{aligned}$$

do not have a unique solution for x, y and z. **[5]**

6 **(i)** The transformation P is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Give a geometrical description of transformation P. **[2]**

(ii) The transformation Q is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Give a geometrical description of transformation Q. **[2]**

(iii) The transformation R is equivalent to transformation P followed by transformation Q. Find the matrix that represents R. **[2]**

(iv) Give a geometrical description of the **single** transformation that is represented by your answer to part **(iii)**. **[3]**

7 It is given that $u_n = 13^n + 6^{n-1}$, where n is a positive integer.

(i) Show that $u_n + u_{n+1} = 14 \times 13^n + 7 \times 6^{n-1}$. **[3]**

(ii) Prove by induction that u_n is a multiple of 7. **[4]**

8 **(i)** Show that $(\alpha - \beta)^2 \equiv (\alpha + \beta)^2 - 4\alpha\beta$. **[2]**

The quadratic equation $x^2 - 6kx + k^2 = 0$, where k is a positive constant, has roots α and β, with $\alpha > \beta$.

(ii) Show that $\alpha - \beta = 4\sqrt{2}k$. **[4]**

(iii) Hence find a quadratic equation with roots $\alpha + 1$ and $\beta - 1$. **[4]**

9 **(i)** Show that $\dfrac{1}{2r-3} - \dfrac{1}{2r+1} = \dfrac{4}{4r^2 - 4r - 3}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=2}^{n} \frac{4}{4r^2 - 4r - 3}.$$ **[6]**

(iii) Show that $\displaystyle\sum_{r=2}^{\infty} \frac{4}{4r^2 - 4r - 3} = \frac{4}{3}$. **[1]**

10 **(i)** Use an algebraic method to find the square roots of the complex number $2 + i\sqrt{5}$. Give your answers in the form $x + iy$, where x and y are exact real numbers. **[6]**

(ii) Hence find, in the form $x + iy$ where x and y are exact real numbers, the roots of the equation

$$z^4 - 4z^2 + 9 = 0.$$ **[4]**

(iii) Show, on an Argand diagram, the roots of the equation in part **(ii)**. **[1]**

(iv) Given that α is the root of the equation in part **(ii)** such that $0 < \arg \alpha < \frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z - \alpha| = |z|$. **[3]**

4725 Further Pure Mathematics 1

1	 $\frac{7}{26}+\frac{17}{26}\text{i}$.	M1 A1 A1 A1	 4 **4**	Multiply by conjugate of denominator Obtain correct numerator Obtain correct denominator
2	**(i)** $\frac{1}{10}\begin{pmatrix}5 & 0\\ -a & 2\end{pmatrix}$ **(ii)** $\begin{pmatrix}3 & -2\\ 2a & 6\end{pmatrix}$	B1 B1 B1 B1	 2 2 **4**	Both diagonals correct Divide by correct determinant Two elements correct Remaining elements correct
3	$n^2(n+1)^2+n(n+1)(2n+1)+n(n+1)$ $n(n+1)^2(n+2)$	M1 A1 A1 M1 A1ft A1	 6 **6**	Express as sum of 3 terms 2 correct unsimplified terms 3rd correct unsimplified term Attempt to factorise Two factors found, ft their quartic Correct final answer a.e.f.
4	 $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$	B1 M1 A1 A1	 4 **4**	State or use correct result Combine matrix and its inverse Obtain **I** or $\mathbf{I}^2$ but not 1 Obtain zero **matrix** but not 0 **S.C. If 0/4, B1 for $\mathbf{AA}^{-1}=\mathbf{I}$**
5	*Either* $4k-4$ $k=1$ *Or*	M1 M1 A1 M1 A1ft M1 A1 M1 A1 A1	 5 **5**	Consider determinant of coefficients of LHS Sensible attempt at evaluating any 3×3 det Obtain correct answer a.e.f. unsimplified Equate det to 0 Obtain k = 1, ft provided all M's awarded Eliminate either x or y Obtain correct equation Eliminate 2nd variable Obtain correct linear equation Deduce that k = 1
6	**(i)** *Either* *Or* **(ii)** **(iii)** $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ **(iv)**	B1 DB1 B1 DB1 B1 DB1 B1 B1 B1B1B1	2 2 2 3 **9**	Reflection, in x-axis Stretch parallel to y-axis, s.f. –1 Reflection, in y = -x Each column correct Rotation, 90° ,clockwise about O **S.C. If (iii) incorrect, B1 for identifying their transformation, B1 all details correct**

7	**(i)** $13^n + 6^{n-1} + 13^{n+1} + 6^n$	B1		Correct expression seen
		M1		Attempt to factorise both terms in (i)
	(ii)	A1	3	Obtain correct expression
		B1		Check that result is true for n =1 (or 2)
		B1		Recognise that (i) is divisible by 7
		B1		Deduce that u_{n+1} is divisible by 7
		B1	4	Clear statement of Induction conclusion
			7	
8	**(i)**	M1		Expand at least 1 of the brackets
		A1	2	Derive given answer correctly
	(ii) $\alpha + \beta = 6k, \alpha\beta = k^2$	B1 B1		State or use correct values
	$\alpha - \beta = (4\sqrt{2})k$	M1		Find value of $\alpha - \beta$ using (i)
		A1		Obtain given value correctly (allow if –6k used)
			4	
	(iii) $\sum \alpha' = 6k$	B1ft		Sum of new roots stated or used
	$\alpha'\beta' = \alpha\beta - (\alpha - \beta) - 1$	M1		Express new product in terms of old roots
	$\alpha'\beta' = k^2 - (4\sqrt{2})k - 1$	A1ft		Obtain correct value for new product
	$x^2 - 6kx + k^2 - (4\sqrt{2})k - 1 = 0$	B1ft	4	Write down correct quadratic **equation**
			10	
9	**(i)**	M1		Use correct denominator
		A1	2	Obtain given answer correctly
	(ii)	M1		Express terms as differences using (i)
		M1		Do this for at least 1st 3 terms
		A1		First 3 terms all correct
		A1		Last 3 terms all correct (in terms or n or r)
	$1 + \frac{1}{3} - \frac{1}{2n-1} - \frac{1}{2n+1}$	M1		Show pairs cancelling
		A1	6	Obtain correct answer, a.e.f.(in terms of n)
	(iii) $\frac{4}{3}$	B1ft	1	Given answer deduced correctly, ft their (ii)
			9	

10	(i) $x^2 - y^2 = 2, 2xy = \sqrt{5}$	M1 A1		Attempt to equate real and imaginary parts Obtain both results a.e.f.
	$4x^4 - 8x^2 - 5 = 0$	M1		Eliminate to obtain quadratic in x^2 or y^2
		M1		Solve to obtain x (or y) values
	$x = \pm\frac{\sqrt{10}}{2}, y = \pm\frac{\sqrt{2}}{2}$	A1		Correct values for both x & y obtained a.e.f.
	$\pm(\frac{\sqrt{10}}{2} + \mathrm{i}\frac{\sqrt{2}}{2})$	A1	6	Correct answers as complex numbers
	(ii) $z^2 = 2 \pm \mathrm{i}\sqrt{5}$	M1 A1		Solve quadratic in z^2 Obtain correct answers
	$z = \pm(\frac{\sqrt{10}}{2} \pm \mathrm{i}\frac{\sqrt{2}}{2})$	M1 A1ft	4	Use results of (i) Obtain correct answers, ft must include root from conjugate
	(iii)	B1ft	1	Sketch showing roots correctly
	(iv)	B1 B1ft B1ft	3 **14**	Sketch of straight line, $\perp$ to α Bisector